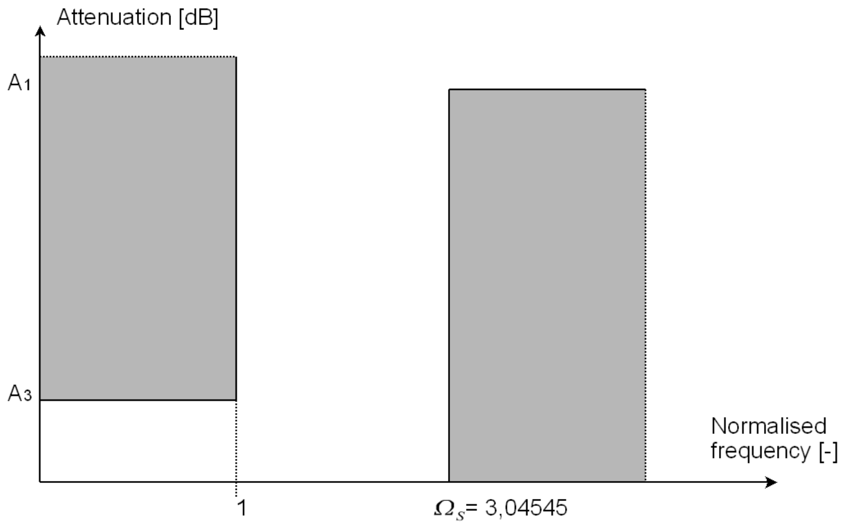
Corrections of Serie 8: Determination of the L and C components of a Chebyshev filter

a) **The specifications of the low-pass filter with respect to the attenuation are given below :**

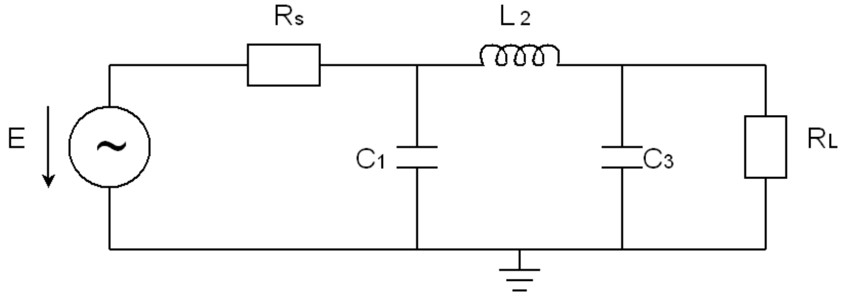
Pass-band : A_3 = 1 dB ; Stop-band: A_1 = 30 dB



Calculate the L and C components of the Chebyshev low-pass filter. The topology with two parallel capacitors and one series inductor is chosen. The resistor Rs of the voltage source is equal to 1 Ohm.

<u>Answer</u>:

The topology with two parallel capacitors and one series inductor for the 3rd order low-pass Chebyshev filter is drawn below.



We calculate the L and C components value by following the procedure described in page 4-22 of the course. We first need to calculate the h and ξ constants value as follows :

$$h = \frac{1}{\varepsilon} + \sqrt{1+\frac{1}{\varepsilon^2}}^{\,1/N} = \frac{1}{0.50885} + \sqrt{1+\frac{1}{0.50885^2}}^{\,1/3} \approx 1.60961$$

and

$$\xi = h - \frac{1}{h} = 1.60961 - \frac{1}{1.60961} \approx 0.9883$$

The normalized values of the components are given by relations (4.26), (4.27) and (4.28) of the course.

Note that the components values are not expressed in F and H but in $F \cdot rad/s$ and $H \cdot rad/s$ because of the frequency normalization (see the x-axis of the specifications with respect to attenuation on page 1). Since R_S is equal to 1 Ω, we have :

$$C_1 = \frac{4\sin\frac{\pi}{2N}}{\xi R_s} = \frac{4\sin\frac{\pi}{6}}{0.9883 \cdot 1} \approx 2.0236\, F \cdot rad/s$$

Setting k = 1 in the first relation (4.27) gives :

$$L_2 = \frac{1}{C_1} \cdot \frac{16\sin\frac{\pi}{6}\sin\frac{3\pi}{6}}{\xi^2 + 2\sin\frac{\pi}{3}^2} = \frac{1}{2.0236} \cdot \frac{16 \cdot 0.5 \cdot 1}{0.9883^2 + 2 \cdot \frac{\sqrt{3}}{2}^2} \approx 0.9941\, H \cdot rad/s$$

Setting k = 1 in the second relation (4.27) gives :

$$C_3 = \frac{1}{L_2} \cdot \frac{16\sin\frac{3\pi}{6}\sin\frac{5\pi}{6}}{\xi^2 + 2\sin\frac{2\pi}{3}^2} = \frac{1}{0.9941} \cdot \frac{16 \cdot 1 \cdot 0.5}{0.9883^2 + 2 \cdot \frac{\sqrt{3}}{2}^2} \approx 2.0236\, F \cdot rad/s$$

Since we finish the filter by a capacitor, we calculate the load resistance with the first relation (4.28) as follows :

$$C_N = \frac{4\sin\frac{\pi}{2N}}{\xi R_L} \Rightarrow R_L = \frac{4\sin\frac{\pi}{2N}}{\xi C_N} = \frac{4\sin\frac{\pi}{6}}{0.9883 \cdot 2.0236} = 1\,\Omega$$

Note that resistances are expressed in Ohm since they are not affected by the frequency normalization.

b)

Calculate the L and C components of the Chebyshev band-pass filter associated to the low pass filter whose components have been calculated in a). It is recalled that this band-pass filter will fulfill the specifications determined in question a) of Serie 7.

<u>Answer</u>:

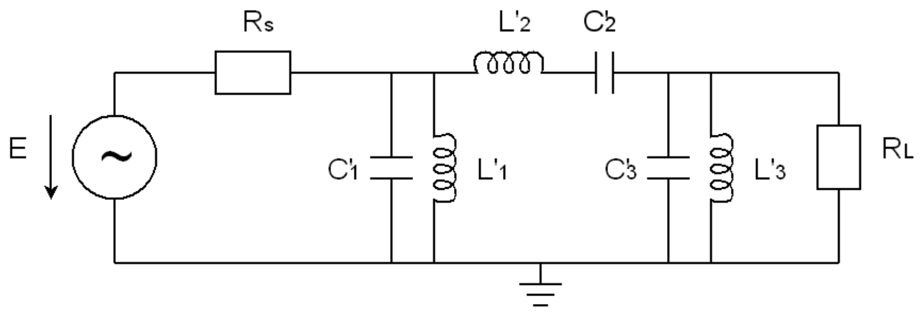
As we have calculated the low-pass filter components values, we can now apply the low-pass to band-pass transformation described in page 4-11. In serie 7, we obtained :

$B = 2\pi(f_4 - f_3) = 2\pi\ 10kHz \approx 62.832\ k\,rad/s$

and

$\omega_0 = 2\pi f_0 = 2\pi\ 454.9725kHz \approx 2.8587\ M\,rad/s$

The pass-band filter is as follows :



where the components values are given by the following relations :

$$C_1 \Rightarrow \begin{cases} C'_1 = \dfrac{C_1}{B} = \dfrac{2.0236\,F\cdot rad/s}{62'832\,rad/s} = 32.2\ \mu F \\ L'_1 = \dfrac{B}{\omega_0^2\cdot C_1} = \dfrac{62.832k\,rad/s}{(2.8587M\,rad/s)^2\cdot 2.0236\,F\cdot rad/s} = 3.7994\ nH \end{cases}$$

$$L_2 \Rightarrow \begin{cases} L'_2 = \dfrac{L_2}{B} = \dfrac{0.9941\,H\cdot rad/s}{62'832\,rad/s} = 15.82\ \mu H \\ C'_2 = \dfrac{B}{\omega_0^2\cdot L_2} = \dfrac{62.832\,k\,rad/s}{(2.8587M\,rad/s)^2\cdot 0.9941\,H\cdot rad/s} = 7.734\ nF \end{cases}$$

$$C_3 \Rightarrow \begin{cases} C'_3 = \frac{C_3}{B} = \frac{2.0236\,F \cdot rad/s}{62'832\,rad/s} = 32.2\mu F \\ L'_3 = \frac{B}{\omega_0^2 \cdot C_3} = \frac{62.832\,k\,rad/s}{(2.8587M\,rad/s)^2 \cdot 2.0236\,F \cdot rad/s} = 3.7994\,nH \end{cases}$$

c)
Draw the gain in dB of the band-pass filter whose components have been calculated in b).

<u>Answer</u>:

The low-pass to band-pass transformation is given by : $s = \frac{p^2+\omega_0^2}{p \cdot B}$

where$s = j\Omega$ and $p = j\omega$. We thus have :

$$j\Omega = \frac{-\omega^2+\omega_0^2}{j\omega \cdot B} = -j\frac{-\omega^2+\omega_0^2}{\omega \cdot B} = -\,j\frac{(\omega_0^2-\omega^2)}{\omega \cdot B} \Rightarrow \Omega = -\,(\frac{\omega_0^2-\omega^2}{\omega \cdot B})$$

So the gain of the filter in dB is expressed as :$20\log|H(j\omega)| = 20\log\frac{1}{\sqrt{1+\varepsilon^2 C_3^2(\Omega)}}$

with $C_3(\Omega) = 4\Omega^3 - 3\Omega$. This gives the following transfer function: